Discrete Mathematics

Course Description

Discrete Mathematics is the study of mathematical systems defined on discrete sets—those with a countable number of elements. It involves the study of objects and ideas that can be divided into separate or discontinuous parts. Most of the mathematics in this area has been discovered during the twentieth century. Among possible topics to be studied are decision making, matrix algebra, graph theory, set theory, Boolean algebra, combinatorics, circuits, discrete probability, recursion, modular arithmetic, network models, sequences and series, mathematical induction, vectors, relations, functions, algorithms, logic, and codes. As well as being of mathematical interest, many of these topics play an important role in information technology and computer science. Discrete mathematics is used every day by computer scientists, computer programmers, electrical engineers, computer engineers, and scientists in the physical, natural, and social sciences.

Depending upon the needs of students and the curricula of other courses offered by districts and/or schools, numerous different syllabi may be written to address these needs by selecting topics in various combinations from the ones listed above, as well as others. One sample syllabus is shown below to provide an example of how selected topics may be organized to meet curricular needs of students.

Recommended Prerequisites

Students entering this course should have successfully completed Algebra 1 (or its equivalent), Algebra 2, and Geometry, having mastered the mathematics standards for those courses.

Sample Course Outline

Alternate topics are included at the end of this sample.

I. Social decision making.
   - A. Use group ranking methods to obtain a single result.
   - B. Determine power indexes for weighted voting systems.
   - C. Solve fair division problems.
     1. Continuous examples.
     2. Discrete examples.
   - D. Solve apportionment problems using a variety of methods.
     1. Hamilton method.
     2. Hill method.
     3. Jefferson method.
     4. Webster method.

II. Graph theory.
   - A. Define basic terms and concepts.
     1. Graphs.

2. Vertices.
3. Edges.
4. Connected graph.
5. Complete graph.
6. Digraphs.
7. Adjacency matrix.
8. Bipartite graph.
9. Planarity.

B. Represent real-world situations using graphs.
C. Find the critical path.
D. Define Euler circuits and paths.
E. Define Hamiltonian circuits and paths.
F. Color graphs.
G. Define and identify trees.
H. Find minimum spanning trees.

III. Matrix theory.
A. Write an adjacency matrix for a graph.
B. Perform basic matrix operations.
C. Use the Leslie model to find population distributions.
D. Use the Leontief input-output model to analyze the flow of goods and services among sectors in an economy.
E. Identify Markov chains.
F. Investigate game theory
1. With a single best strategy.
2. Without a single best strategy.

IV. Counting techniques.
A. Define combinations and permutations.
B. Evaluate expressions indicating permutations or combinations.
C. Construct and examine Pascal's triangle.
D. Use the addition and multiplication principles to solve problems.
E. Define and calculate conditional probabilities.
F. Calculate expected value.

V. Iteration and recursive relations.
A. Determine the terms of a sequence.
1. Explicit.
2. Recursive.

B. Evaluate sums of geometric series.
1. Finite.
2. Infinite.

C. Solve problems using recursion and mixed recursion.
D. Solve exponential growth problems.
E. Solve finance and population problems.
F. Use the method of finite differences for finding closed-form solutions.
G. Construct cobweb diagrams.
H. Define fractals.
I. Investigate chaos theory.

VI. Linear programming.
   A. Solve real-world problems using linear programming.
   B. Use technology to solve linear programming problems.
VII. Mathematical induction.
VIII. Logic.
   A. Identify a statement as existential, universal, or neither.
   B. Write and use counterexamples and properties.
   C. Negate statements.
   D. Determine the truth of a conditional statement.
   E. Write a logical expression to describe a simple network.
   F. Find the contrapositive, converse, or inverse of a conditional statement.
   G. Use logic to prove or disprove a statement.
   H. Determine the validity or invalidity of arguments.

Depending upon the needs of students and the curricula of other courses offered by districts and/or schools, one or more of the following may be taught instead of selected topics in the sample syllabus shown above.

Alternate Topics and Related Objectives

I. Vectors.
   A. Solve problems involving two-dimensional and three-dimensional vectors.
      1. Geometric representation of vectors.
      2. Geometric representation of complex numbers.
      3. Algebraic representation of vectors.
      4. Vectors and parametric equations.
      5. Dot product.
      6. Cross product.
      7. Vectors and planes.
II. Sequences and series.
   A. Solve problems involving series and sequences.
      1. Arithmetic sequences.
      2. Geometric sequences.
      3. The *n*th term of an arithmetic or geometric sequence.
      4. Recursive definitions of sequences.
      5. Arithmetic series.
      6. Geometric series.
      7. The sum of *n* terms of an arithmetic or geometric series.
      8. Infinite geometric series.
      9. Sums of special geometric series.
      10. Fibonacci series.
   B. Write proofs of the validity of summation formulas.
      1. Summation formula by Gauss.
      2. Mathematical induction.
III. Matrices and determinants.
   A. Solve problems using matrices.

1. Matrix addition.
2. Scalar multiplication.
3. Matrix multiplication.
4. Inverse matrices.

B. Solve problems using matrices and determinants.
1. Systems of equations.
2. Transformations using matrices.
3. Systems with augmented matrices.

IV. Graphs and their applications.

A. Special functions.
1. Piecewise functions.
2. Greatest integer functions.

B. Venn diagrams.

C. Linear programming.

D. Paths and circuits.
1. Multigraphs.
2. Loops and parallel edges.
3. Paths, simple paths, distinct cycles, and their lengths.
4. Euler paths or circuits.

E. Coloring a graph.
1. Chromatic number for a graph.
2. Welsh-Powell algorithm to color graph.

F. Directed graphs.

V. Recurrence.

A. Values and initial conditions.

B. First-order linear difference equations.

C. Second-order homogeneous linear difference equations.

D. Searching and sorting algorithms.

VI. Arithmetic.

A. Bases (decimal, binary, octal, hexadecimal).

B. Conversions between bases.

C. Arithmetic in non-decimal bases.

D. Computer representation of numbers.

E. Computer arithmetic.

VII. Propositional logic.

A. Propositions.

B. Binary connectives.

C. Truth tables.

D. Laws of logic.

E. Validity of arguments.

F. Predicate logic.

G. Quantifiers.

VIII. Boolean algebra.

A. Axiomatic definition.

B. Simplifying Boolean expressions.

C. Digital circuits.

D. Disjunctive normal form.
E. Karnaugh maps.
F. Logic gates.